NORTH CAROLINA STATE UNIVERSITY

Department of Mechanical and Aerospace Engineering

MAE 521 Robust Control with Convex Methods

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Project: Aircraft Pitch Pointing and Vertical Translation Control

REPORT

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**Introduction**

This part is the brief introduction

**Dynamic model**





The output , where *nzp* is normal acceleration at the pilot’s station.



So .

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**Approaches and solutions**

**Part 1:** examine the controllability and observability of the MINO and SISO systems.

1. For MIMO system, generate matrix of Mc and Mo and check their rank. If the rank of Mc is not full, then the MIMO system is uncontrollable. Similar with Mo, if the rank of Mo is not full, then the MIMO system is unobservable.



Rank (Mc) =5



Rank (Mo) =5.

So the MIMO system is both controllable and observable.

1. SISO system: with the same expression of Mc and Mo, test controllability and observability for each combination of SISO system. Since the controllability is only related to the A matrix and input B, then examine the rank of Mc for both input separately.

Input 1:

, rank (Mc1) =4

Input 2:

, rank (Mc2) =4

Mc for both inputs are not full rank. It’s similar that observability of one system is only related to it’s A matrix and output C. So examine the rank of Mo for each output

Output1

, rank (Mo1) =3

Output 2:

, rank (Mo2) =3

Output 3:

, rank (Mo3) =4

All Mo for 3 outputs are not full rank. So each SISO system is not controllable and observable.

Table 1. Controllability and observability for SISO system with each combination of input and output

|  |  |  |  |
| --- | --- | --- | --- |
|  | Output 1 | Output 2 | Output 2 |
| Input 1 | Uncontrollable, unobservable | Uncontrollable, unobservable | Uncontrollable, unobservable |
| Input 2 | Uncontrollable, unobservable | Uncontrollable, unobservable | Uncontrollable, unobservable |

**Part 2:** Examine the open-loop system response to an impulse disturbance on angle of attack and step input change on both elevator and flaperon deflections, separately.

Use matlab to simulate two kinds of input signal into the system. Fig. shows the responses to these two kinds of input signal. From the Fig. 1, it could be obtained that pitch attitude state has no responses to both input signals. The responses from normal acceleration changed greater than pitch rate. The step input of elevator and flaperon deflections influenced responses more than one impulse disturbance on angle of attack state.

|  |  |
| --- | --- |
| a) | b) |

Fig 1. Open-loop system responses to a) impulse disturbance on angle of attack; b) step input change on elevator and flaperon deflections.

Part 3 and part 4: Design a full-state feedback system , where the feedback matrix *F* is 2x5 feedback gain matrix. Examine the SVD of the feedback system and comment on its min-max behavior

Requirements:

1. Minimize the steady-state error and pay close attention to the cost of control;
2. try to shape the eigenvector directions for the first and the third states as follows;

,where X denotes ‘don’t care’ entries

1. limit the normal acceleration output within 3.0 g.

Solution:

First trial

Step 1: examine the eigenvalues of the open loop system.

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Notice that the eigenvalues of the open-loop system are not all negative, so the open-loop system is not stable.

Step 2: choose eigenvalues and eigenvectors for closed-loop.

Choose as the closed-loop poles. Use MIMO eigenvector assignment, set P=, Q=

Use equation for each P and Q, then get feedback gain F= ,result.

Step 3: evaluate the steady state error:

e=

step 4: get SVD and plot, find output of normal accelaraion is about 5g, greater than 3.0 g

second trial: increase eigenvalue

same eigenvectors, obtain F, cost

steady-state error, bad than first, output good

third trial: add scaling matrix before r

shape c into 2\*2, get K

steady-state error, but normal accelation large

fourth trial, shape c by choosing, get K’

steady-state error, but normal acceleration overshoot

for the safety of the pilot, choose the second trial feedback gain.

Part 5: set the elements in A matrix related to state to be zero, same with B and C. So new system will be

Set the F= so that outputs will not response to the signal of input 2

Let the eigenvalues equals to those in second trial to minimize the sacrifice of performance

**Evaluation**

PART 5

Since the control of flaperon was disabled, so the output has variantion by only changing input2. So at first trial, set the full state feed back gain F=[1,2,3,4,5;0,0,0,0,0] and test the system, Use input 2 is sinusoid with zero input 1. However, the responses still changing with the time varying input. Then set elements related to input 2 in B to be zero. Then we got a time unvarying response to sinusoid input.

For least porfomance, it will be better to let the transfer function of new system getting closer to old one. So use

Part 6 – Design an observer for the system (either full-state or reduced order) using output measurements of pitch rate and pitch attitude. Make sure that your observer dynamics is at least five times faster than your system dynamics.

A reduced order is designed below. Since the second and third state could be measured, the state-space model need to be manipulated as below







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The pole of fast mode of dynamic system is -18, and the observer dynamics should not less than five times the fast mode for system dynamics. So choose the pole of observer to be -90,-91,-92.

Appendix-A Matlab code for step 1—controllability and observability for MIMO and SISO system

clear all;

% Define state

A=[-1.3,0.98,0,-0.165,-0.248;42.81,-0.785,0,-17.3,-1.58;1.25,0.007,0,0.165,0.248;0,0,0,-18,0;0,0,0,0,-18];

B=[0,0;0,0;0,0;18,0;0,18];

C=[0,1,0,0,0;46.5,-0.256,0,-4.25,4.15;0,0,1,0,0];

D=[0,0;0,0;0,0];

[U,V]=eig(A)

% observable,controllable for MIMO

Mc=[B,A\*B,A^2\*B,A^3\*B,A^4\*B];

rank(Mc);

Mo=[C',A'\*C',A'^2\*C',A'^3\*C',A'^4\*C'];

rank(Mo);

%%

% Mc for input 1

B1=B(:,1);

C1=C(1,:);

Mc1=[B1,A\*B1,A^2\*B1,A^3\*B1,A^4\*B1];

rank(Mc1); %=4

% Mc for input 2

B2=B(:,2);

Mc2=[B2,A\*B2,A^2\*B2,A^3\*B2,A^4\*B2];

rank(Mc2); %=4

% Mo for output 1

Mo1=[C1',A'\*C1',A'^2\*C1',A'^3\*C1',A'^4\*C1'];

rank(Mo1); %=3

% Mo for output 2

C2=C(2,:);

Mo2=[C2',A'\*C2',A'^2\*C2',A'^3\*C2',A'^4\*C2'];

rank(Mo2); %=3

% Mo for output 3

C3=C(3,:);

Mo3=[C3',A'\*C3',A'^2\*C3',A'^3\*C3',A'^4\*C3'];

rank(Mo3); %=4

Appendix-B: Matlab code for step 2—responses to impulse disturbance on angle and attack, and step input on elevator and flaperon deflections

sys=ss(A,B,C,D);

t=0:0.001:10;

tt=transpose(t);

% impulse disturbance

figure(1)

BB=[B,[1;0;0;0;0]];

D=[D,[0;0;0]]

sys2=ss(A,BB,C,D);

dis=[1;zeros(size(transpose(0:0.001:9.999)))]

uu=[zeros(size(tt)) zeros(size(tt)) dis];

[Y,TT]=lsim(sys2,uu,t)

plot(TT,Y(:,1),'k',TT,Y(:,2),'K--',TT,Y(:,3),'K-.')

xlabel('Time(s)')

ylabel('Responses')

title('Open-loop system responses to an impulse disturbance on angle of attack')

legend('pitch rate','normal accelaration','pitch attitude')

% step input

figure(2)

u=[ones(size(tt)) ones(size(tt))];

[X,T]=lsim(sys,u,t);

plot(T,X(:,1),'k',T,X(:,2),'K--',T,X(:,3),'K-.')

legend('pitch rate','normal accelaration','pitch attitude')

xlabel('Time(s)')

ylabel('Responses')

title('Open-loop system responses to step input on elevator and flaperon deflection')